Distributional Semantics and Embeddings

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Words as Vectors

- Context Weighting
- Word2Vec
- Other Kinds of Static Embeddings
- Semantic Properties of Embeddings
- **Bias and Embeddings**
- **References and Credits**

Humans infer meaning from the context

McDonald & Ramscar (2001)

• What is a *wampimuk*?

"He filled the **wampimuk**, passed it around and we all drunk some." A container for drinks?

"We found a little, hairy **wampimuk** *sleeping behind the tree."* A little animal?

• What other words could occur in this context?

"He filled the ___, passed it around and we all drunk some."

- cup
- goblet
- bottle

"We found a little, hairy ____ sleeping behind the tree."

- monkey
- squirrel
- rabbit

- Words that occur in similar contexts tend to have similar meanings
- Firth (1957): "You shall know a word by the company it keeps."
- Vectors as word representations to describe the properties of each word
- Distributional statistics: core part of language technology leverage large amounts of unlabeled data to learn about (rare) words

Constructing a Distributional Semantic Model

understand t	he	causes of these	climate	anomalies is in	nportant	not only for historical
information on how Earth 's		climate	has changed over thous		usands and thousands of	
oceans are	also	threatened by	climate	change , with	temper	ature rises of

- Context: words next to \boldsymbol{w} in a fixed-size window
- Use contexts of all observations of ${\boldsymbol w}$ to build a representation of ${\boldsymbol w}$

• Collecting context words of **w** (content words only):

causes	1
anomalies	1
important	1
Earth	1
changed	1
threatened	1
change	1

Context Design

- Size of the context window
- Differentiate: left-side context and right-side context
- All words in a windos vs. selected words in a window, lemma vs. inflected forms
- POS-coded: change_{NOUN} vs. change_{VERB}
- Structured information: dependency information (e.g. SUBJ, OBJ, PP relations)
 - dependency-filtered: consider only particular relations postman: bite, dog: bite
 - dependency-linked: also keep information of the depency path postman: obj-of-bite, dog: subj-of-bite
- ⇒ Task-specific

Words as Vectors: Word Dimensions and Similarity

	aardvark	 computer	data	result	pie	sugar	
cherry	0	 2	8	9	442	25	
strawberry	0	 0	0	1	60	19	
digital	0	 1670	1683	85	5	4	
information	0	 3325	3982	378	5	13	

Figure 6.6 Co-occurrence vectors for four words in the Wikipedia corpus, showing six of the dimensions (hand-picked for pedagogical purposes). The vector for *digital* is outlined in red. Note that a real vector would have vastly more dimensions and thus be much sparser.

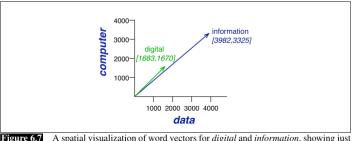


Figure 6.7 A spatial visualization of word vectors for *digital* and *information*, showing just two of the dimensions, corresponding to the words *data* and *computer*.

Cosine to Measure Similarity

- Measure similarity between vectors of the words v and w
- Common similarity measure: cosine of the angle between the vectors

$$\operatorname{cosine}(\mathbf{v}, \mathbf{w}) = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}| |\mathbf{w}|} = \frac{\sum_{i=1}^{N} v_i w_i}{\sqrt{\sum_{i=1}^{N} v_i^2} \sqrt{\sum_{i=1}^{N} w_i^2}}$$
(6.10)

Values range from 1 (= same direction) to 0 (for orthogonal vectors)
 -1 for opposite directions, but we don't get this with non-negative frequencies

	pie	data	computer
cherry	442	8	2
digital	5	1683	1670
information	5	3982	3325

$$cos(cherry, information) = \frac{442 * 5 + 8 * 3982 + 2 * 3325}{\sqrt{442^2 + 8^2 + 2^2}\sqrt{5^2 + 3982^2 + 3325^2}} = .018$$

$$cos(digital, information) = \frac{5 * 5 + 1683 * 3982 + 1670 * 3325}{\sqrt{5^2 + 1683^2 + 1670^2}\sqrt{5^2 + 3982^2 + 3325^2}} = .996$$

 \Rightarrow information is closer to digital than to cherry

Cosine to Measure Similarity

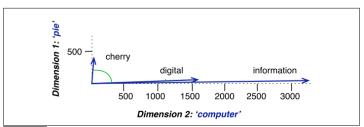


Figure 6.8 A (rough) graphical demonstration of cosine similarity, showing vectors for three words (*cherry, digital,* and *information*) in the two dimensional space defined by counts of the words *computer* and *pie* nearby. The figure doesn't show the cosine, but it highlights the angles; note that the angle between *digital* and *information* is smaller than the angle between *digital* and *information* is smaller than the angle between *digital* is smaller; the cosine has its maximum (1) when the angle between two vectors is smallest (0°) ; the cosine of all other angles is less than 1.

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- Raw context counts → scores taking into account the relevance of an observed context
- Association measures: higher weight to context words highly associated with target word
- Co-occurrence with a frequent context element is less informative: the less frequent a context element is → the higher the weight should be for the observed co-occurrence
- Different measures, for example point-wise mutual information

• Intuition: how much more do two words occur in a corpus than we would expect at chance?

$$PMI(w,c) = \log_2 \frac{P(w,c)}{P(w)P(c)}$$
(6.17)

- Numerator: how often we observed the word and the context together
- Denominator: how often we expect the two words to co-occur Probability of two independent events both occurring: product of the probabilities of the events
- Ratio: estimate of how much more the two words co-occur than we expect by chance

Point-wise Mutual Information

- PMI scores lie between $-\infty$ and $+\infty$
- Negative PMI means that words are co-occurring less than we expect by chance
- Negative PMI values tend to be unreliable unless the corpora are enormous

Unclear whether scores of "unrelatedness" are meaningful

• Positive point-wise mutual information: replace negative values with 0

$$PPMI(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P(c)}, 0)$$
(6.18)

Point-wise Mutual Information: Example

	computer	data	result	pie	sugar	count(w)
cherry	2	8	9	442	25	486
strawberry	0	0	1	60	19	80
digital	1670	1683	85	5	4	3447
information	3325	3982	378	5	13	7703
count(context)	4997	5673	473	512	61	11716

Figure 6.10 Co-occurrence counts for four words in 5 contexts in the Wikipedia corpus, together with the marginals, pretending for the purpose of this calculation that no other words/contexts matter.

Thus for example we could compute PPMI(information,data), assuming we pretended that Fig. 6.6 encompassed all the relevant word contexts/dimensions, as follows:

$$P(\text{w=information, c=data}) = \frac{3982}{11716} = .3399$$

$$P(\text{w=information}) = \frac{7703}{11716} = .6575$$

$$P(\text{c=data}) = \frac{5673}{11716} = .4842$$

$$PPMI(\text{information,data}) = \log_2(.3399/(.6575 * .4842)) = .0944$$

Point-wise Mutual Information: Example

	p(w,context)					p (w)
	computer	data	result	pie	sugar	p(w)
cherry	0.0002	0.0007	0.0008	0.0377	0.0021	0.0415
strawberry	0.0000	0.0000	0.0001	0.0051	0.0016	0.0068
digital	0.1425	0.1436	0.0073	0.0004	0.0003	0.2942
information	0.2838	0.3399	0.0323	0.0004	0.0011	0.6575
p(context)	0.4265	0.4842	0.0404	0.0437	0.0052	
Figure 6 11	Replacing the c	ounts in Fig	66 with ic	int probabi	lities chowin	a the marginals

Figure 6.11 Replacing the counts in Fig. 6.6 with joint probabilities, showing the marginals in the right column and the bottom row.

	computer	data	result	pie	sugar	
cherry	0	0	0	4.38	3.30	
strawberry	0	0	0	4.10	5.51	
digital	0.18	0.01	0	0	0	
information	0.02	0.09	0.28	0	0	

Figure 6.12 The PPMI matrix showing the association between words and context words, computed from the counts in Fig. 6.11. Note that most of the 0 PPMI values are ones that had a negative PMI; for example PMI(*cherry, computer*) = -6.7, meaning that *cherry* and *computer* co-occur on Wikipedia less often than we would expect by chance, and with PPMI we replace negative values by zero.

Point-wise Mutual Information: Rare Words

- PMI has the problem of being biased toward infrequent events very rare words tend to have very high PMI values
- Reduce bias toward low frequency: change the computation for P(c)
- Function $P_{\alpha}(c)$: raise the probability of c to the power of α

$$PPMI_{\alpha}(w,c) = \max(\log_2 \frac{P(w,c)}{P(w)P_{\alpha}(c)}, 0)$$
(6.21)

$$P_{\alpha}(c) = \frac{count(c)^{\alpha}}{\sum_{c} count(c)^{\alpha}}$$
(6.22)

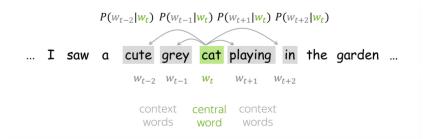
• In practice: α = 0.75 leads to good results

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 What we want: put information about contexts into vectors count-based methods: long, sparse vectors; dimensions corresponding to |V|

- Word2Vec: Learn vectors by teaching them to predict contexts Mikolov et al. 2013(a); Mikolov et al. 2013(b)
- Word2Vec's parameters are word vectors that are optimized for a certain objective
- Objective: make vectors "know" about the context of it word train vectors to predict possible contexts
- Hypothesis: "know" about context \rightarrow "know" about meaning

Word2Vec: Idea



- Main idea
 - all words in the vocabulary V are represented by a vector
 - go over corpus with a sliding window: one central word and context words to the left and right
 - calculate the probability of a context word given the center word using the similarity of word the vectors
 - adjust vectors to maximize the probabilities

Objective Function: Negative Log-Likelihood

 For each position t = 1, ..., T, predict context words in a fixed-size window of size m given the central word w_t:

$$\underbrace{ \text{Likelihood}}_{\theta \text{ is all variables}} L(\theta) = \prod_{t=1}^{T} \prod_{\substack{t=1 \\ j \neq 0}} P(w_{t+j} \mid w_t; \theta)$$

The objective function is the average negative log-likelihood

sometimes called a cost or loss function

The objective function $J(\theta)$ is the (average) negative log likelihood:

$$J(\theta) = -\frac{1}{T}\log L(\theta) = -\frac{1}{T}\sum_{\substack{t=1 \ -m \leq j \leq m \\ j \neq 0}}^{T}\log P(w_{t+j} \mid w_t; \theta)$$

Minimizing objective function ⇔ Maximizing predictive accuracy

Figures from web.stanford.edu/class/cs224n/slides/cs224n-spr2024-lecture01-wordvecs1.pdf

Calculate the Probabilities

- How to calculate $P(w_{t+j}|w_t, \theta)$?
- For each word w we have two vectors
 - $-v_w$ when it is a central word
 - u_w when it is a context word
- Softmax to map arbitrary values x_i to a probability distribution p_i

$$softmax(x_i) = \frac{exp(x_i)}{\sum\limits_{j=i}^{n} exp(x_j)}$$

max: the largest x_i will have the largest probability p_i soft: all probabilities are non-zero

• $P(o|c) = \frac{exp(u_o^T v_c)}{\sum_{w \in V} exp(u_w^T v_c)}$

dot product: measure similarity of o and c larger dot product = larger probability

normalize over vocabulary to get probability distribution

where and c = central word and o = outside word

Word2Vec: Word and Context Vectors

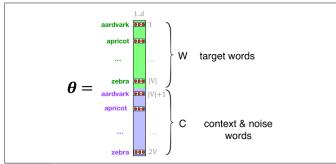


Figure 6.13 The embeddings learned by the skipgram model. The algorithm stores two embeddings for each word, the target embedding (sometimes called the input embedding) and the context embedding (sometimes called the output embedding). The parameter θ that the algorithm learns is thus a matrix of 2|V| vectors, each of dimension *d*, formed by concatenating two matrices, the target embeddings **W** and the context+noise embeddings **C**.

Train: Gradient Descent, one Word at a Time

- Parameters θ : vectors v_w and u_w for all words in |V|
- Optimize training objective via gradient descent (with learning rate α)

$$\theta^{new} = \theta^{old} - \alpha \nabla_{\theta} J(\theta)$$

- Make updates one at a time: each update is for a single pair of center word and a context word
- Loss function $J(\theta) = -\frac{1}{T} \log L(\theta) = -\frac{1}{T} \sum_{\substack{t \\ i\neq 0}}^{T} \log P(w_{t+j}|w_t, \theta) = \frac{1}{T} \sum_{\substack{t=1 \\ t=1}}^{T} \sum_{\substack{-m \leq j \leq m \\ j\neq 0}}^{T} J_{t,j}(\theta)$

• For the center word w_t , the loss contains a distinct term $J_{t,j}(\theta) = -logP(w_{t+j}|w_t, \theta)$ for each of its context words w_{t+j}

... I saw a cute grey cat playing in the garden ...

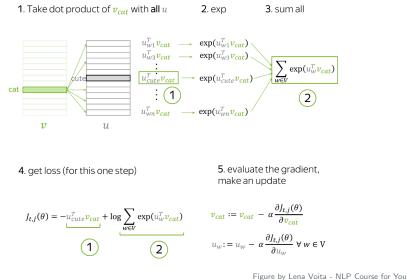
• Loss for the central word *cat* and the context word *cute*:

$$J_{t,j}(\theta) = -\log P(cute|cat) = -\log \frac{\exp u_{cute}^T v_{cat}}{\sum\limits_{w \in Voc} \exp u_w^T v_{cat}} = -u_{cute}^T v_{cat} + \log \sum\limits_{w \in Voc} \exp u_w^T v_{cat}.$$

- Parameters at this step:
 - vectors for central words: only w_{cat}
 - vectors context words: all u_w
- \Rightarrow These parameters will be updated

Figure by Lena Voita - NLP Course for You

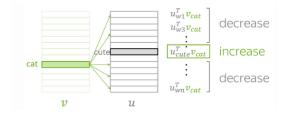
Example: Gradient Update



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Example: Gradient Update

- Making an update to minimize $J_{t,j}(\theta)$
 - \rightarrow increase the similarity (i.e. dot product) of v_{cat} and u_{cute}
 - \rightarrow decrease the similarity between v_{cat} and u_w for all other words w



- Why decrease similarity between *v_{cat}* and all other words? some of them might be valid contexts as well
- Updates for all context words for all target words
 → averaged over all updates, the vectors will learn the distribution

Figure by Lena Voita - NLP Course for You

Faster Training: Negative Sampling

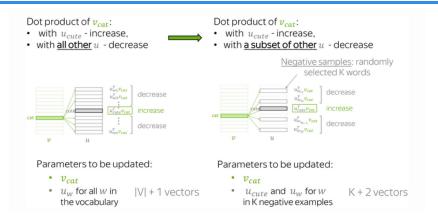
- For each pair of central word and its context word: update all vectors for context words
- Highly inefficient: each step is proportional to the vocabulary size
- Don't consider all context vectors in V, but randomly sample just a few negative examples

... lemon, a [tablespoon of apricot jam, a] pinch ... c1 c2 w c3 c4

positive examples +		negative examples -				
W	cpos	w	cneg	w	Cneg	
apricot	tablespoon	apricot	aardvark	apricot	seven	
apricot	of	apricot	my	apricot	forever	
apricot	jam	apricot	where	apricot	dear	
apricot	a	apricot	coaxial	apricot	if	

• Word2Vec uses more negative than positive examples

Faster Training: Negative Sampling



- Increase similarity between u_{cat} and u_{cute}
- Decrease similarity between u_{cat} and subset of k negative examples
- Large corpus: on average, all updates will update each vector sufficient number of times

Faster Training: Negative Sampling

• The new loss function:

$$J_{t,j}(\theta) = -\log \sigma(u_{cute}^T v_{cat}) - \sum_{w \in \{w_{i_1}, \dots, w_{i_K}\}} \log \sigma(-u_w^T v_{cat})$$

- $w_1, ..., w_k$ are the k negative examples
- Sigmoid function to map the dot product into a probability:

$$\sigma(x) = \frac{1}{1 + exp^{-x}} \qquad \text{Note: } \sigma(-x) = 1 - \sigma(x)$$

• Write the loss also as

$$J_{t,j}(heta) = -\log \sigma(u_{cute}^T v_{cat}) - \sum_{w \in \{w_{i_1}, \ldots, w_{i_K}\}} \log(1 - \sigma(u_w^T v_{cat})).$$

Figure by Lena Voita - NLP Course for You

Word2Vec: Word and Context Vectors

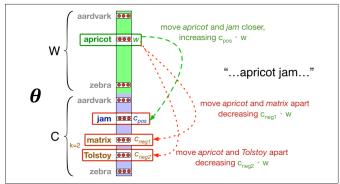


Figure 6.14 Intuition of one step of gradient descent. The skip-gram model tries to shift embeddings so the target embeddings (here for *apricot*) are closer to (have a higher dot product with) context embeddings for nearby words (here *jam*) and further from (lower dot product with) context embeddings for noise words that don't occur nearby (here *Tolstoy* and *matrix*).

- Each word has only few "true" contexts: randomly chosen words are likely "negative" (= not true) contexts
- Noise words are chosen according to their empirical distribution
- Modified to sample less frequent words more often: weighted unigram frequency p_α(w)

sample *the* with probability $p_{\alpha}(the)$ sample *aardvark* with probability $p_{\alpha}(aardvark)$

- In practice, set $\alpha = 0.75$ and use $P_{\alpha}(w) = \frac{count(w)^{\alpha}}{\sum_{w'} count(w')^{\alpha}}$
- We did the same for PPI

Word2Vec: Variants

- Skip-Gram: predicts context words given the central word.
 Skip-Gram with negative sampling is the most popular approach
- CBOW (Continuous Bag-of-Words): predicts the central word from the sum of context vectors (= bag of words)

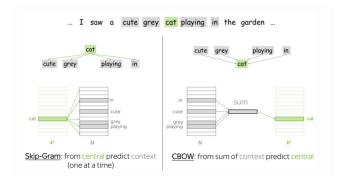


Figure by Lena Voita - NLP Course for You

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Fasttext

- Word2Vec: no good way to handle unknown words and sparsity
 - unknown: occurs in test data, but unseen in training data
 - sparsity (for example in morphologically rich languages): some word forms only rarely seen in training data \rightarrow unreliable representation
- Fasttext Bojanowski et al. (2017)
- Enriches word vectors with subword information
- Subword models: represent each word as itself and a bag of constituent n-grams
- For example word with n = 3: <word> and <wo, wor, ord, rd>
- Learn a skipgram embedding for each constituent n-gram
- Unknown words: represented by the sum of the constituent n-grams

• GloVe: Gobal Vectors

Pennington et al. (2014)

- Based on global corpus statistics: count-based methods to measure association between word *w* and context *c*
- GloVe controls the influence of rare and frequent words
- Loss function is designed such that
 - rare events are penalized
 - very frequent events are not over-weighted

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Visualizing Embeddings

- Visualizing embeddings is important in order to understand and apply such models
- Simplest way to visualize the meaning of a word: list the most similar words according to cosine similarity



• Sometimes, the nearest neighbors according to this metric reveal rare but relevant words that lie outside an average human's vocabulary

Figure from https://nlp.stanford.edu/projects/glove/

Visualizing Embeddings

- Clustering: show a hierarchical representation of which words are similar to others
- Probably most common method: project *d* dimensions of a word to two dimensions, for example by means of t-SNE (t-Distributed Stochastic Neighbor Embedding)



T-SNE visualisation of word embeddings generated using 19th century literature

Figure by Siobhán Grayson, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=64541584 https://en.wikipedia.org/wiki/T-distributed_stochastic_neighbor_embedding

- Size of context window: generally between 1 and 10 words on each side of the target word
- Shorter context windows: capture functional and syntactic similarities information comes from nearby words → most similar words tend to be semantically similar with the same part-of-speech: *Poodle, Pitbull, Rottweiler*
- Longer context window: topically related words: dog, bark, leash

Analogy/Relational Similarity

• Models like word2vec or GloVe can solve analogy problems $\overrightarrow{king} - \overrightarrow{man} + \overrightarrow{woman}$ is vector close to \overrightarrow{queen} $\overrightarrow{Paris} - \overrightarrow{France} + \overrightarrow{Italy}$ is vector close to \overrightarrow{Rome}

The embedding model thus seems to be extracting **representations of relations** like *male* – *female*, or *capital* – *city-of*, or even *comparative* – *superlative*

- But ... it doesn't always work
 - just returning morpological variants: $\overrightarrow{cherry} - \overrightarrow{red} + \overrightarrow{potato}$ returns $\overrightarrow{potatoes}$ instead of \overrightarrow{brown}
 - embedding spaces perform well if the task involves frequent words, small distances, and certain relations (relating countries/capitals or verbs/nouns with inflected forms), but not so well for other relations

Analogy/Relational Similarity

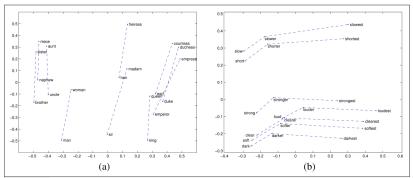


Figure 6.16 Relational properties of the GloVe vector space, shown by projecting vectors onto two dimensions. (a) $\overline{\text{king} - \text{man}} + \overline{\text{woman}}$ is close to $\overline{\text{queen}}$. (b) offsets seem to capture comparative and superlative morphology (Pennington et al., 2014).

Figure from Jurafsky and Martin

Embeddings and Historical Semantics

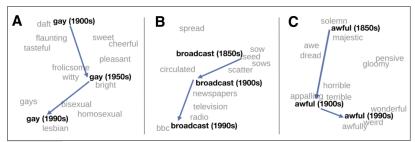


Figure 6.17 A t-SNE visualization of the semantic change of 3 words in English using word2vec vectors. The modern sense of each word, and the grey context words, are computed from the most recent (modern) time-point embedding space. Earlier points are computed from earlier historical embedding spaces. The visualizations show the changes in the word *gay* from meanings related to "cheerful" or "frolicsome" to referring to homosexuality, the development of the modern "transmission" sense of *broadcast* from its original sense of sowing seeds, and the pejoration of the word *awful* as it shifted from meaning "full of awe" to meaning "terrible or appalling" (Hamilton et al., 2016b).

Figure from Jurafsky and Martin

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Bias and Embeddings

- Embeddings reproduce the implicit biases and stereotypes that were latent in the training data
- For example, gender stereotypes :

computer programmer - man + woman → homemaker father:doctor::mother:nurse

- But also racism, ...
- Bias amplification: gendered terms become *more* gendered in embedding space
- Debiasing: transform the embedding space such that gender stereotypes are removed, but definitional gender is preserved. Still an open problem ...

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- This lecture is mostly based on and contains content from
 - Chapter 6 in Speech and Language Processing, Jurafsky and Martin (2024)
 - Lena Voita's NLP Course for You

https://lena-voita.github.io/nlp_course/word_embeddings.html#w2v_idea

• Some slides took inspiration from a presentation of Marco Baroni and Gemma Boleda

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